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The c-functions of non-commutative Yang-Mills theory from holography

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Abstract: In this paper we study non-commutative Yang-Mills theory (NCYM) through its gravity dual. First it is shown that the gravity dual of an NCYM with self-dual θ-parameters has a lagrangian in the form of 5-dimensional dilatonic gravity. Then we use the de-Boer-Verlinde-Verlinde formalism for holographic renormalization group flows to calculate the coefficient functions in the Weyl anomaly of the NCYM at low energies under the assumption of potential dominance, and show that the C-theorem holds true in the present case.

Keywords: AdS/CFT Correspondance, Non-Commutative Geometry.

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Contents

1. Introduction

Yang-Mills theory on a non-commutative space [1, 2], or simply non-commutative Yang-Mills theory (NCYM), has recently received increasing attention in string theory community. A few years ago, coordinates for coincident D-branes were shown to naturally promote to matrices [3], signaling the relevance of non-commutative gauge theory [4]. Later, NCYM was shown to actually appear in the D-string solution to the IIB matrix model [5] and in various string/M(atrix) theory compactification with constant NS-NS B-background [6]–[10]. This is not too surprising, because for a single D-brane in a background with constant gauge field-strength or rank-two anti-symmetric B-tensor, some appropriate limit should lead to a situation similar to that of a particle in the lowest Landau level, where the guiding-center coordinates are known to be non commuting. Indeed in a recent seminal paper, among other things, Seiberg and Witten [11] have explicitly identified the precise limit in string theory for NCYM to work, which is similar to the limit in M-Theory for discrete light cone quantization of Matrix theory to work [12]. In this way, NCYM arises as a new limit in string theory, providing a new probe to non-perturbative effects in string/M(atrix) theory.

In this paper we study NCYM by exploring its supergravity dual. By now it is widely believed that gauge theory is dual to a certain limit of string theory [13, 14]; in particular, type-IIB supergravity on an anti-de Sitter background, say of five dimensions, can be used to describe a large-N supersymmetric Yang-Mills (SYM) theory on the 4-dimensional boundary, which is known to be a conformal field theory (CFT). One important test of this AdS/CFT correspondence is the holographic derivation of the quantum Weyl anomaly in the $D = 4$, $\mathcal{N} = 4$, SU(N) SYM from the generally covariant boundary counter-terms in the classical action of its bulk AdS gravity dual [15], with central charges reproducing the expected large-N behavior. It seems natural to extend this correspondence between gauge theory and gravity to NCYM. The supergravity backgrounds with non-vanishing B-fields that are supposed to be dual to NCYM have been recently suggested in refs. [16, 17, 18]. Furthermore, it was observed in ref. [19] that these supergravity duals can be derived from the Seiberg-Witten relations [11] between closed and open string moduli, by assuming the running string tension is a simple power function of energy. This observation suggests that one should try to use the supergravity duals to explore the running behavior of NCYM.

It is known that NCYM is no longer conformally invariant, because of the length scale associated with a non-vanishing B-background. Thus, one expects that the "central charges" of NCYM, defined as the coefficients in its Weyl anomaly, should run as a function of the energy scale. It is interesting to calculate these functions, the so-called c-functions, and to see whether they obey a generalization of the famous C-theorem [20] in two dimensions, that asserts the c-function is always monotonically increasing with the energy scale. The consistent coupling of NCYM to a curved background is not known yet, so it is not possible at this moment to directly calculate the Weyl anomaly of NCYM on the field theory side. The goal of the present paper is to study the c-functions in a holographic manner through the supergravity dual, thus providing constraints and shedding light on the problem of consistently coupling NCYM to a curved background.

The method we are going to use to calculate the holographic Weyl anomaly is the Hamilton-Jacobi approach to the 5-dimensional bulk gravity developed by de Boer, Verlinde and Verlinde [21]. In this approach an analogue of the first-order Callan-Symanzik equations for the 4-dimensional dual field theory on the boundary can be derived from the bulk Hamilton-Jacobi equations. The key point is to interpret the Hamilton-Jacobi functional as the quantum effective action in the dual field theory resulting from integrating out the matter degrees of freedom coupled to the boundary background gravity. Then from it one can derive the holographic Weyl anomaly and c-functions. Moreover, in the de Boer-Verlinde-Verlinde formalism there are dilaton-like scalar fields in 5-dimensional gravity. The radial profile of these fields in the bulk represents the renormalization group (RG) running of certain coupling constants in the dual field theory on the boundary. To apply this formalism, one needs to show that the NCYM's gravity dual given in [17] for the full 10-dimensional IIB background really has a 5-dimensional dilatonic gravity lagrangian after dimensional reduction. In this paper we will show that this is indeed the case for an NCYM with self-dual θ-parameters, corresponding to isotropic non-commutativity, whose gravity dual has a self-dual B-background, such that the B-field does not explicitly appear in the action for the dilaton-gravity sector after dimensional reduction to 5 dimensions.

The paper is organized as follows. In section 2 we show that after dimensional reduction from 10-dimensional IIB supergravity, the gravity dual of an NCYM with self-dual non-commutativity parameters has a 5-dimensional lagrangian in the form of a dilatonic gravity. In section 3 the de Boer-Verlinde-Verlinde formalism for holographic renormalization group flows is adopted to calculate the c-functions of the NCYM at low energies under the assumption of potential dominance. In section 4, we show that the c-functions defined in section 2 transform as vectors on the dilaton-space as the beta function does. The final section, section 5, is devoted to conclusions and discussions. In appendix A we show how to generalize the de Boer-Verlinde-Verlinde formalism to the non-canonical form of dilatonic gravity.

2. Dilaton gravity dual of non-commutative Yang-Mills

In contrast to the gravity dual of ordinary Yang-Mills theory, the supergravity dual of NCYM involves turning on non-trivial scalar and various r-form (anti-symmetric r-tensor) backgrounds with a radial profile [17]. One may wonder if there exists a 5-dimensional gravity action from which the equations of motion dimensionally reduced from 10-dimensional IIB supergravity can be derived. We will show that there is indeed such a 5-dimensional dilatonic gravity action, at least for the case with self-dual B-backgrounds.

The bosonic action for type-IIB supergravity in ten dimensions in the Einstein frame is

$$I_{10} = \frac{1}{2\,\kappa_{10}^2}\int d^{10}z\sqrt{\det G}\,\bigg[R - \frac{1}{2}\,(\partial\phi)^2 - \frac{1}{2}\,e^{2\phi}(\partial\chi)^2 - \frac{1}{2\cdot 3!}\,e^{-\phi}\,H_3^2 - \\ - \frac{1}{2\cdot 3!}\,e^{\phi}F_3^2 - \frac{1}{4\cdot 5!}\,F_5^2\bigg]\,, \tag{2.1}$$

where ϕ is the dilaton, χ the RR scalar and the form strengths are defined as

$$\begin{aligned} H_3 &= dB_2\,,\\ F_3 &= dA_2 - \chi\,H_3\,,\\ F_5 &= dA_4 - \frac{1}{2}\,A_2\wedge H_3 + \frac{1}{2}\,B_2\wedge F_3\,. \end{aligned} \tag{2.2}$$

Here B_2 and A_2 are respectively the NS-NS and RR 2-form potentials, A_4 are the RR 4-form potential and the 5-form strength F_5 is self-dual, that is

$$*F_5 = -i\,F_5\,, \tag{2.3}$$

where $*$ is the 10-dimensional Hodge dual.

In this paper, we only consider self-dual B-backgrounds, together with the following conditions motivated by supersymmetry [26]:

$$\chi - i\,e^{-\phi} = i\,c\,, \qquad F_3 = i\,cH_3\,,$$
$$B_{01} = B_{23}\,, \qquad A_{01} = A_{23}\,, \tag{2.4}$$

where c is a real constant.

These conditions are consistent with the equations of motion for scalars and 2-form potentials. Moreover, they make the contributions to the energy-momentum tensor from the NS-NS and RR sector cancel each other, except the one from self-dual 5-form strength. The Einstein equations and the Gauss law for the 5-form strength thus form a closed set of equations:

$$R_{MN} = T^F_{MN}\,, \tag{2.5}$$
$$\partial_M\left(\sqrt{\det G}\,F^{MNPQR}\right) = 0\,, \tag{2.6}$$

where $T^F_{MN} = \frac{1}{4\cdot 4!}\,F_{MPQRS}\,F_N^{\;PQRS}$ is the energy-momentum tensor of the 5-form strength. Once the solution to this set is known, the other fields can be solved from their equations of motion.

We now perform dimensional reduction by using the following ansatz for the 10-dimensional metric:

$$ds^2 = G_{MN}(z)\,dz^M\,dz^N = \hat{g}_{mn}(x)\,dx^m\,dx^n + \ell^2\,\Phi(x)\,\bar{g}_{ab}(y)\,dy^a\,dy^b\,, \tag{2.7}$$

with the indices $\{m,n\}$ running on the reduced 5-dimensional manifold $\mathcal{M}$, and indices $\{a,b\}$ on a prescribed 5-sphere with x-dependent radius $\sqrt{\ell^2\Phi(x)}$, where ℓ^2 is the typical length scale of $\mathcal{M}$ related to the D3-brane charge (or the 5-form flux), and the Jordan-Brans-Dicke scalar Φ turns out to be the dilaton in the reduced theory.

Using this ansatz we can solve the Gauss law equation (2.6) for the 5-form strength, and the solution is

$$F_{mnopq} = i\,4\,\ell^{-1}\,\Phi^{-5/2}\sqrt{\det\hat{g}}\,\epsilon_{mnopq}\,, \tag{2.8}$$

where the ϵ-symbol is equal to 1 (-1) for even (odd) permutations of $0,1,2,3,r$, and to 0 otherwise.

The components of the corresponding energy-momentum tensor are

$$T^F_{mn} = -\frac{4}{\ell^2}\,\Phi^{-5}\,\hat{g}_{mn}\,, \qquad T^F_{ab} = \frac{4}{\ell^2}\,\Phi^{-4}\,\bar{g}_{ab}\,, \qquad T^F_{ma} = 0\,. \tag{2.9}$$

The 10-dimensional Einstein equation (2.5) then decomposes into

$$R_{mn} = \hat{R}_{mn} - \frac{5}{2}\,\Phi^{-1}\,\hat{\nabla}_m\,\hat{\nabla}_n\Phi + \frac{5}{4}\,\Phi^{-2}\,\hat{\nabla}_m\Phi\,\hat{\nabla}_n\Phi = -\frac{4}{\ell^2}\,\Phi^{-5}\,\hat{g}_{mn}\,, \tag{2.10}$$

$$R_{ab} = \left(\frac{4}{\ell^2} - \frac{1}{2}\,\hat{\nabla}^2\Phi - \frac{3}{4}\,\Phi^{-1}\left(\hat{\nabla}\Phi\right)^2\right)\bar{g}_{ab} = \frac{4}{\ell^2}\,\Phi^{-4}\,\bar{g}_{ab}\,, \tag{2.11}$$

$$R_{ma} = \frac{1}{2}\,\Phi^{-1}\,\partial_m\Phi\,\bar{g}^{bc}\left(\bar{\nabla}_c\,\bar{g}_{ab} - \bar{\nabla}_a\,\bar{g}_{bc}\right) = 0\,, \tag{2.12}$$

where $\hat{R}_{mn}$ is the Ricci tensor of the metric $\hat{g}_{mn}$, while $\hat{\nabla}$ and $\bar{\nabla}$ the covariant derivative with respect to $\hat{g}_{mn}$ and $\bar{g}_{ab}$,respectively.

Given the prescribed 5-sphere metric $\bar{g}_{ab}$, eq. (2.11) reduces to a single equation of motion for the dilaton Φ, and eq. (2.12) is just an identity because of the metricity condition.

It turns out that this reduced set of equations of motion (2.10) and (2.11) for $\hat{g}_{mn}$ and Φ can be derived from the following 5-dimensional action for a dilatonic gravity

$$I_5 = \frac{V_5}{2\,\kappa_{10}^2} \int_{\mathcal{M}} d^5x \, \sqrt{\det \hat{g}}\, \Phi^{5/2} \left[\hat{R} + 5\,\Phi^{-2} (\hat{\nabla}\Phi\,)^2 + \ell^{-2} \left(20\,\Phi^{-1} - 8\,\Phi^{-5}\right) \right], \quad (2.13)$$

where V_5 is the volume of the unit 5-sphere. This action reduces to the familiar action for AdS gravity if we set $\Phi = 1$.

To bring the gravity action to the canonical Einstein-Hilbert form we need to do the following Weyl transformation

$$\hat{g}_{mn} = \Phi^{-5/3}\, g_{mn}\,, \quad (2.14)$$

and the corresponding new 5-dimensional action is

$$I_5^{EH} = \frac{V_5}{2\,\kappa_{10}^2} \int_{\mathcal{M}} d^5x \, \sqrt{\det g} \left[R - \frac{10}{3}\,\Phi^{-2} (\nabla\Phi)^2 + \ell^{-2}\,\Phi^{-8/3} \left(20 - 8\Phi^{-4}\right) \right], \quad (2.15)$$

where the unhatted quantities are with respect to the new 5-dimensional metric g_{mn}.

This action was derived before from the gauged supergravity point of view in a different context [27]. Our above discussions establish that the proposed gravity dual [17] of an NCYM with isotropic non-commutativity has a 5-dimensional dilatonic gravity description given by the action (2.13) or (2.15).

One may feel odd at first sight that in the 5-dimensional reduced dilatonic gravity dual, the 2-form field that specifies the non-commutativity in the original boundary Yang-Mills theory does not show up explicitly. The puzzle is resolved by noticing that the self-duality conditions (2.4) place strong restrictions on the holographic profile of the dilaton Φ to make it dependent on the asymptotic value of the 2-form field. This is most easily seen from the full expression of Maldacena and Russo's solution [17] (in the near horizon limit):

$$\begin{aligned} ds_E^2 &= \ell^2\, r^2\, \Phi \Big\{ \Phi^{-2} \left(dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + r^{-4}\, dr^2 + r^{-2}\, d\Omega_5^2 \Big\}\,, \\ \Phi &= \left(1 + a^4 r^4\right)^{1/2}\,, \qquad F_{0123r} = i\,4\,\ell^{-1} \Phi^{-5/2} \sqrt{\det \hat{g}} = i\,4\,\ell^4\, r^3\, \Phi^{-4}\,, \\ B_{01} &= B_{23} = \sqrt{g_s}\, a^2\, r^4\, \Phi^{-2}\,, \qquad A_{01} = A_{23} = -i\,\frac{a^2\,\ell^2}{\sqrt{g_s}}\, r^4\, \Phi^{-2}\,, \\ e^{2\phi} &= g_s^2\, \Phi^{-2}\,, \qquad \chi = i\,\frac{a^4\, r^4}{g_s}\,, \end{aligned} \quad (2.16)$$

where g_s is the string coupling in the IR limit $r = 0$, and $a^2 = \sqrt{4\pi\alpha'^2 g_s N}/B^\infty$, $\ell^2 = \alpha'\sqrt{4\pi N}$ with α' the string scale, N the D3-brane charge and B^∞ the boundary value of B-field at $r = \infty$. Note that the world-volume (specified by the directions $0, 1, 2, 3$) quantities have been properly re-scaled as in [17], and also that with a self-dual 2-form background (or non-commutativity parameters) the world volume metric for the NCYM remains isotropic.

As one can see, the profile of Φ is chosen so that the solutions of NSNS and RR scalars in (2.16) satisfy the first equation of the self-duality conditions (2.4), which would not hold for an arbitrary dilaton profile. Following the proposal of [17], we then conclude that the dilatonic gravity (2.15) with the specific dilaton radial profile given in (2.16) is the holographic dual of the NCYM with isotropic non-commutativity. Moreover, we can read the 5-dimensional background metrics from (2.16), that is

$$\hat{g}_{mn}\, dx^m\, dx^n = \ell^2\, r^{-2} \Phi\, dr^2 + \ell^2\, r^2\, \Phi^{-1} dx_\parallel^2 \tag{2.17}$$

for the action I_5, and

$$g_{mn}\, dx^m\, dx^n = \ell^2\, r^{-2} \Phi^{8/3} dr^2 + \ell^2\, r^2\, \Phi^{2/3} dx_\parallel^2 \tag{2.18}$$

for the action I_5^{EH}, where $x_\parallel$ represents the longitudinal coordinates and r is called the holographic coordinate which is the energy scale from the field theory point of view.

3. Holographic RG flow of NCYM in self-dual B-background

The existence of a consistent effective 5-dimensional dilatonic gravity allows us to generalize a counter-term generating algorithm in the AdS/CFT correspondence, known as the holographic renormalization group (RG) flow that is determined by the dilaton profile [25]. The dilaton is interpreted as an effective coupling running with the energy scale in the dual field theory. If the dilaton has a constant radial profile, the theory reduces to pure AdS gravity with the holographic dual a CFT having a vanishing beta function. When the dilaton has a non-trivial radial profile, the holographic Callan-Symanzik RG equations have been constructed by de Boer, Verlinde and Verlinde in an elegant formalism using the standard Hamilton-Jacobi theory [21, 22]. The c-functions in the Weyl anomaly can then be calculated.

The essence of the de Boer-Verlinde-Verlinde formalism is the observation that though the equations of motion of the 5-dimensional supergravity is of second order, the evolution equation of its on-shell action S, derived from the standard Hamilton-Jacobi theory, is of first order and takes the usual form of the Callan-Symanzik equations, therefore S can be interpreted as the 4-dimensional quantum effective action after integrating out all the matter degrees of freedom coupled to the background gravity.

According to the holographic interpretation of the gravity dual, a preferred radial coordinate in the bulk gravity can be selected out as representing the energy scale of the dual field theory. For simplicity, we choose the "tempora" gauge for 5-dimensional metric

$$g_{mn}\, dx^m\, dx^n = d\rho^2 + \gamma_{\sigma\nu}(\rho, x)\, dx^\sigma\, dx^\nu\,, \tag{3.1}$$

where ρ is the holographic radial coordinate.

For the metric on a boundary screen located at the radial position ρ, we can further separate out the holographic coordinate dependence:

$$\gamma_{\sigma\nu}(\rho, x) = \mu^2(\rho)\, \bar{\gamma}_{\sigma\nu}(x)\,, \tag{3.2}$$

where $\bar{\gamma}_{\sigma\nu}$ is the background geometry seen by the dual field theory at some fundamental scale, and the warp factor μ^2 is the overall length scale on the boundary screen. According to the holographic UV/IR relation [24], μ stands for the energy scale of the boundary QFT, and we define the beta function for the dilaton Φ by

$$\beta \equiv \mu\, \frac{d\Phi}{d\mu}\,, \tag{3.3}$$

which can be easily calculated once the 5-dimensional metric and the dilaton profile are given.

For example, the proposed gravity dual (2.16) of NCYM with isotropic commutativity has the dilaton profile

$$\Phi = \left(1 + a^4 r^4\right)^{1/2}\,, \tag{3.4}$$

and the energy scale can be read from the defining metric (2.18), (3.1) and (3.2):

$$\mu = \ell\, r\, \Phi^{1/3} = \frac{\ell}{a}\, \Phi^{1/3} \left(\Phi^2 - 1\right)^{1/4}\,, \tag{3.5}$$

and the resulting beta function from (3.3) and (3.5) is

$$\beta = \frac{6\, \Phi\, (\Phi^2 - 1)}{5\, \Phi^2 - 2}\,. \tag{3.6}$$

Note that μ is a monotonically increasing function of Φ and r, so the UV limit $r \to \infty$ corresponds to $\mu \to \infty$ and $\Phi \to \infty$, and the IR limit $r \to 0$ to $\mu \to 0$ and $\Phi \to 1$.

To develop the Hamilton-Jacobi theory, we shall cast the 5-dimensional dilatonic gravity action into the canonical formalism using the above metric:

$$\begin{aligned} I &= \frac{1}{2\,\kappa_5^2} \int_{\mathcal{M}} d^5x\, \sqrt{\det g}\, \left[R + \frac{1}{2}\, G(\Phi)(\nabla\Phi)^2 + V(\Phi)\right] \\ &\equiv \frac{1}{2\,\kappa_5^2} \int d\rho\, L\,, \end{aligned} \tag{3.7}$$

$$L = \int d^4x\, \sqrt{\det \gamma}\, \left[\pi_{\sigma\nu}\, \dot{\gamma}^{\sigma\nu} + \Pi\, \dot{\Phi} - \mathcal{H}\right]. \tag{3.8}$$

Here $\dot{}$ denotes the derivative with respect to ρ, and the canonical momenta and the hamiltonian density are defined by

$$\pi_{\sigma\nu} \equiv \frac{1}{\sqrt{\det\gamma}}\frac{\delta L}{\delta\dot{\gamma}^{\sigma\nu}}\,, \qquad \Pi \equiv \frac{1}{\sqrt{\det\gamma}}\frac{\delta L}{\delta\dot{\Phi}}\,, \tag{3.9}$$

$$\mathcal{H} \equiv \frac{1}{3}\pi^2 - \pi_{\sigma\nu}\,\pi^{\sigma\nu} + \frac{\Pi^2}{2\,G} - \mathcal{L}\,, \tag{3.10}$$

$$\mathcal{L} \equiv \mathcal{R} + \frac{1}{2}\,G\gamma^{\sigma\nu}\,\partial_\sigma\Phi\,\partial_\nu\Phi + V\,, \tag{3.11}$$

with $\mathcal{R}$ the Ricci scalar of the boundary metric $\gamma_{\mu\nu}$. Note that $\int d^4x\,\mathcal{L}$ is the action dimensionally reduced from (3.7).

The defining equations of canonical momenta (3.9) can be inverted to obtain the first-order flow equations

$$\dot{\gamma}_{\sigma\nu} = 2\,\pi_{\sigma\nu} - \frac{2}{3}\,\gamma_{\sigma\nu}\,\pi^\sigma_\sigma\,, \tag{3.12}$$

$$\dot{\Phi} = \frac{\Pi}{G}\,. \tag{3.13}$$

These equations will be helpful in solving the resulting Hamilton-Jacobi equation.

In the canonical formulation of the gravity theory, the hamiltonian density $\mathcal{H}$ gives rise to a constraint $\mathcal{H} = 0$, imposed upon the canonical variables:

$$\frac{1}{3}\pi^2 - \pi_{\sigma\nu}\,\pi^{\sigma\nu} + \frac{\Pi^2}{2\,G} = \mathcal{R} + \frac{1}{2}\,G\,\gamma^{\sigma\nu}\,\partial_\sigma\Phi\,\partial_\nu\Phi + V\,. \tag{3.14}$$

We then introduce the Hamilton-Jacobi functional S with a properly assumed form, and see if we can derive first-order evolution equations for terms in S. With the hint of the AdS/CFT correspondence, one interprets S as the quantum effective action of the dual field theory after integrating out the matter degrees of freedom coupled to the background gravity, which is assumed of the usual form on a curved space:

$$\begin{aligned} S[\gamma,\Phi] &= S_{EH}[\gamma,\Phi] + \Gamma[\gamma,\Phi]\,,\\ S_{EH}[\gamma,\Phi] &= \int d^4x\,\sqrt{\det\gamma}\left[Z(\Phi)\,\mathcal{R} + \frac{1}{2}\,M(\Phi)\,\gamma^{\sigma\nu}\,\partial_\sigma\Phi\,\partial_\nu\Phi + \mathrm{U}(\Phi)\right]. \end{aligned} \tag{3.15}$$

S_{EH} is the tree level renormalized action which is similar in structure to the lagrangian density $\mathcal{L}$, and Γ contains the higher-derivative and non-local terms.

In the Hamilton-Jacobi theory, the canonical momenta are related to the Hamilton-Jacobi functional S by

$$\pi_{\sigma\nu} = \frac{1}{\sqrt{\det\gamma}}\frac{\delta S}{\delta\gamma^{\sigma\nu}}\,, \qquad \Pi = \frac{1}{\sqrt{\det\gamma}}\frac{\delta S}{\delta\Phi}\,. \tag{3.16}$$

With these relations and the interpretation of S as the effective quantum action, the quantum average of the boundary stress tensor $\langle T_{\sigma\nu}\rangle$ and that of the gauge invariant

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operator $\langle O_\Phi \rangle$ to which Φ couples can be related to Γ by

$$\langle T_{\sigma\nu} \rangle = \frac{2}{\sqrt{\det\gamma}} \frac{\delta\Gamma}{\delta\gamma^{\sigma\nu}}, \qquad \langle O_\Phi \rangle = \frac{1}{\sqrt{\det\gamma}} \frac{\delta\Gamma}{\delta\Phi}. \tag{3.17}$$

The factor of two is determined from the Hamilton-Jacobi equation by requiring the correct proportionality to the beta-function term in the Weyl anomaly:

$$\langle T^\sigma_\sigma \rangle = \beta \langle O_\Phi \rangle - c\,\mathcal{R}_{\sigma\nu}\mathcal{R}^{\sigma\nu} + d\mathcal{R}^2, \tag{3.18}$$

where β is the beta function defined in (3.3), and c and d are the c-functions.

Substituting (3.15) into (3.16) we obtain the explicit form of the canonical momenta, which will be helpful in solving the Hamilton-Jacobi equation (3.14),

$$\begin{aligned}\pi_{\sigma\nu} &= \frac{1}{2}\langle T_{\sigma\nu}\rangle + Z\,\mathcal{R}_{\sigma\nu} + \left(\frac{M}{2} - Z''\right)\partial_\sigma\Phi\,\partial_\nu\Phi - Z'\nabla_\sigma\nabla_\nu\Phi - \\ &\quad - \frac{1}{2}\gamma_{\sigma\nu}\left[Z\,\mathcal{R} + \left(\frac{M}{2} - 2\,Z''\right)(\nabla\Phi)^2 - 2\,Z'\nabla^2\Phi + U\right], \end{aligned} \tag{3.19}$$

$$\Pi = \langle O_\Phi \rangle + Z'\mathcal{R} - \frac{M'}{2}(\nabla\Phi)^2 - M\,\nabla^2\Phi + U', \tag{3.20}$$

where $'$ denotes the derivative with respect to Φ and the covariant derivatives here are with respect to $\gamma_{\sigma\nu}$.

To derive the evolution equations for terms in S_{EH}, we insert the expansion (3.19) and (3.20) into (3.14), and solve the resulting Hamilton-Jacobi equation by equating terms on both sides with the same functional form. With this procedure we get from the potential term,

$$\frac{U^2}{3} + \frac{U'^2}{2\,G} = V, \tag{3.21}$$

and from the curvature term,

$$\frac{U}{3}\,Z + \frac{U'}{G}\,Z' = 1. \tag{3.22}$$

Note both are first-order evolution equations.

Moreover, combining the second-order curvature terms and the first-order terms in the quantum average $\langle T^\sigma_\sigma \rangle$ and $\langle O_\Phi \rangle$, we can obtain the expression for the Weyl anomaly which is of the form of (3.18), with the c-functions given by

$$c = \frac{6\,Z^2}{U}, \qquad d = \frac{2}{U}\left(Z^2 + \frac{3\,Z'^2}{2\,G}\right). \tag{3.23}$$

We can rewrite the curvature part of (3.18) in terms of the Euler density $\mathcal{E}$, the Weyl density $\mathcal{W}$ and the Ricci scalar squared as follows:

$$c\,\mathcal{R}_{\sigma\nu}\,\mathcal{R}^{\sigma\nu} + d\mathcal{R}^2 = \frac{c}{2}(\mathcal{E} - \mathcal{W}) + \left(d - \frac{c}{3}\right)\mathcal{R}^2, \tag{3.24}$$

where

$$\mathcal{E} = \mathcal{R}^2 - 4\,\mathcal{R}_{\sigma\nu}\mathcal{R}^{\sigma\nu} + \mathcal{R}_{\sigma\nu\lambda\delta}\,\mathcal{R}^{\sigma\nu\lambda\delta}\,, \qquad \mathcal{W} = \frac{1}{3}\,\mathcal{R}^2 - 2\,\mathcal{R}_{\sigma\nu}\,\mathcal{R}^{\sigma\nu} + \mathcal{R}_{\sigma\nu\lambda\delta}\,\mathcal{R}^{\sigma\nu\lambda\delta}\,. \quad (3.25)$$

Note that $\mathcal{E}$ is a topological density, and $\mathcal{W}$ is an invariant under Weyl transformations, so that the combination $\mathcal{E} - \mathcal{W}$ is invariant up to a total derivative under Weyl transformations. However, the $\mathcal{R}^2$ term is not a Weyl invariant, whose presence signals the non-conformal nature of NCYM when $c \neq 3d$, as shown later.

Because of the nonlinearity, it is difficult to solve U from (3.21). We, however, can solve it from the flow equations by substituting (3.19) and (3.20) into (3.12) and (3.13). Assuming that the theory is at sufficiently low energy scale compared to the cutoff so that the potential term dominates, it then yields

$$U = \frac{6\,\dot{\mu}}{\mu}\,, \quad (3.26)$$

$$\beta \equiv \mu\,\frac{d\Phi}{d\mu} = \frac{6\,U'}{G\,U}\,. \quad (3.27)$$

Clearly the effective cosmological constant U is over-determined by three eqs. (3.21), (3.26) and (3.27), the consistency of the solutions among them will imply the validity of the formalism and the assumption of potential dominance, which reminds us that the theory is at sufficiently low energy scale.

On the other hand, the effective inverse Newton constant Z will be determined by (3.22) up to an integration constant given by the initial conditions. There are also equations determining M in the kinetic term from the input of U and Z; however, we omit them since our interest is the c-functions which are independent of M, and it is easy to show that M can be consistently solved from the Hamilton-Jacobi equation.

Having the formalism at hand, we are ready to calculate the running behavior of the quantum effective action S for the NCYM from its dilatonic gravity dual defined by (2.15) and (2.18). The beta function for Φ has been given in (3.6). Compare (2.15) and (3.7), we have

$$G(\Phi) = \frac{-20\,\Phi^{-2}}{3}\,, \qquad V(\Phi) = \ell^{-2}\Phi^{-8/3}\left(20 - 8\,\Phi^{-4}\right). \quad (3.28)$$

With these data and eq. (3.6) for the beta function, we find the solutions for the effective cosmological constant U from the three equations mentioned above agree with each other, all giving

$$U = \frac{2\,\Phi^{-10/3}}{\ell}\left(5\Phi^2 - 2\right). \quad (3.29)$$

The running behavior of the effective inverse Newton constant is then determined from (3.22) and is given by

$$Z = \frac{\ell}{6}\,\Phi^{-2/3}\left(\Phi^2 + 2\right) + Z_0\,\Phi^{-2/3}\left(\Phi^2 - 1\right)^{-1/2}. \quad (3.30)$$

Note that the second term blows up in the IR limit $\Phi \to 1$ if $Z_0 \neq 0$, which will violate the assumption of potential dominance at low energy scale; and thus we are forced to set $Z_0 = 0$.

Finally, from (3.23) the resulting c-functions are (for $Z_0 = 0$)

$$c = \frac{\ell^3}{12} \frac{\Phi^2 \left(\Phi^2 + 2\right)^2}{5\,\Phi^2 - 2}\,, \qquad d = \frac{\ell^3}{60} \frac{\Phi^2 \left(\Phi^4 + 8\Phi^2 + 6\right)}{5\,\Phi^2 - 2}\,. \tag{3.31}$$

In the above equations (3.28) to (3.31), the profile of the dilaton Φ is given by eq. (3.4). Like the beta function, the c-functions are monotonically increasing with Φ (and thus with μ) for $\Phi \geq 1$. This is a generalization of Zamolodchikov's C-theorem [20] in two dimensions that the c-functions are always monotonically increasing with the energy scale; so one may say that *the C-theorem holds true in the present case*. Away from the IR limit, $\Phi > 1$ and the ratio $c/d \neq 3$, differing from the one ($c/d = 3$) for ordinary Yang-Mills theory in the usual AdS/CFT correspondence with $\Phi \equiv 1$, which is the IR limit of the NCYM.

4. c-functions as vectors on the Φ-space

In the section 2 we have seen that the form of the 5-dimensional dilatonic gravity action, dimensionally reduced from 10-dimensional supergravity as the dual of NCYM, is not unique. We have obtained two such actions, one given by (2.15) with a canonical Einstein-Hilbert term for gravity and the other (2.13) of a non-canonical form; they are related to each other by a Weyl transformation (2.14). In the section 3, we have chosen to work with the canonical form of the action (2.15). One may wonder what are the resulting beta and c-functions if we work with the non-canonical action (2.13). The answer for the beta function is straightforward: from its definition (3.3), it should transform as a vector on the Φ-space which can be thought as the coupling constant space of NCYM. To be explicit, let us call the energy scale parameter μ_q,[1] for the non-canonical gravity in contrast to the parameter μ defined for the canonical one. These two quantities are related to each other by the Weyl transformation (2.14), which through (3.2) leads to

$$\mu_q = \Phi^{-5/6} \mu\,. \tag{4.1}$$

From this, the beta functions in the two cases are related by

$$\beta \equiv \mu \frac{d\Phi}{d\mu} = \Omega\, \mu_q \frac{d\Phi}{d\mu_q} \equiv \Omega\, \beta_q\,, \tag{4.2}$$

$$\Omega \equiv \frac{\mu}{\mu_q} \frac{d\mu_q}{d\mu} = \frac{3}{5\,\Phi^2 - 2}\,. \tag{4.3}$$

[1] The subscript q will be used to specify the non-canonical counterparts of the quantities defined in section 3; the same convention will be also adopted in appendix A.

Though the beta function has clear geometric meaning by its definition, it is not clear if the c-functions have also the geometric meaning as vectors on the Φ-space. To answer this question, we need to generalize the de-Boer-Verlinde-Verlinde formalism to the non-canonical action. The generalization is straightforward but tedious, we will leave the details to appendix A. The resulting c-functions turn out to be

$$c_q = \Omega^{-1} c\,, \qquad d_q = \Omega^{-1} d \tag{4.4}$$

and are thus vectors on the Φ-space. Note that (4.4) is true as long as the integration constants for Z_q and Z are set to equal, that is

$$Z_q = \Phi^{5/3} Z = \frac{\ell}{6}\,\Phi\left(\Phi^2 + 2\right) + \frac{Z_0\,\Phi}{\sqrt{\Phi^2 - 1}}\,. \tag{4.5}$$

Now that the c-functions have a geometric interpretation, it would be interesting to see if the C-theorem may have a generic geometric origin. This issue has been explored in the recent works [28] on the gravity side. We leave this problem for NCYM for future study.

5. Conclusions and discussions

Since Maldacena and Russo proposed the supergravity dual of NCYM, not much has been done along this line. In this paper, we first pointed out that the gravity dual of NCYM with isotropic non-commutativity has a consistent 5-dimensional action in the form of a dilatonic gravity, which enables us to adopt the holographic RG flow approach to investigate the physics on the dual field theory side, generalizing the usual AdS/CFT correspondence.

We adopted the de Boer-Verlinde-Verlinde formalism to evaluate the c-functions at low energies, under the assumption of potential dominance, and found that the C-theorem holds true in the present case. The ratio of the two-coefficient fucntions in the Weyl anomaly away from the IR limit is different from that in ordinary Yang-Mills theory, indicating the non-conformal nature of the NCYM. All of these were seen from the dual gravity side. To examine these phenomena directly inside the NCYM itself is worthwhile, especially because the perturbative techniques of non-commutative field theory seem to have become matured in a series of recent works [29, 30].

The calculations of Weyl anomaly and boundary counter-terms for the boundary conformal theory from the AdS gravity have been performed in many different ways [15, 23], they all agree to each other. Not much similar efforts have been spent for the non-commutative cases. Besides the method adpoted in this paper, there is an alternative approach [31] by generalizing the method of Henningson and Skenderis [15] to dilaton gravity which applies only to asymptotically AdS spacetime. However, as pointed out in the second paper of ref. [31], the NCYM dual at hand, corresponding to eq. (57) there, has not asymptotic AdS region in UV. It would be interesting to see whether an improvement of their approach can be applied to the NCYM dual.

Although we have defined the c-functions from its gravity dual by calculating the Weyl anomaly, we still lack a general understanding from the field theory side. It has been an issue of defining sensible c-functions in 4 dimensions and there is an on-going debate about the validity of a general 4-dimensional C-theorem [32]. In section 4, we clarified the nature of c-functions on the coupling constant space, and showed they are indeed vectors on the coupling constant space as the beta function. We hope this geometric understanding will help in constructing a geometric realization of the C-theorem in 4 dimensions.

Up to now, we have only considered the supergravity background with self-dual B-field configurations. It would be interesting to consider more general B-backgrounds, which will correspond to NCYM with anisotropic non-commutativity. The 5-dimensional gravity dual will then be a dilaton gravity coupled to the 2-form potentials, and we need to generalize the de Boer-Verlinde-Verlinde formalism to include the dynamics of 2-form potentials, which may help us to understand more about the physics of NCYM from its gravity dual.

A. Generalization of de Boer-Verlinde-Verlinde formalism for non-canonical gravity

We start with the non-canonical action (2.13) and cast it into the ADM form as done for the canonical one:

$$I_5 = \frac{V_5}{2\,\kappa_{10}^2}\int d^5x\,\sqrt{\det\hat{g}}\left[X_q(\Phi)\hat{R} + \frac{1}{2}\,G_q(\Phi)\left(\hat{\nabla}\Phi\right)^2 + V_q(\Phi)\right] \tag{A.1}$$

$$\equiv \frac{V_5}{2\,\kappa_{10}^2}\int dr\,L_q\,, \tag{A.2}$$

where X_q, G_q and V_q can be read from (2.13).

Decompose the metric into the warped form

$$\hat{g}_{mn}\,dx^m\,dx^n = N^2\,d\rho^2 + \gamma_{\sigma\nu}(\rho,x)\,dx^\sigma\,dx^\nu\,,\quad N=\pm 1\,, \tag{A.3}$$

(with N the lapse function). Using the identity

$$\hat{R} = -2\,\nabla_m\left(n^m\nabla_n n^n\right) + \mathcal{R} - \left(\mathcal{K}_{\sigma\nu}\,\mathcal{K}^{\sigma\nu} - \mathcal{K}^2\right), \tag{A.4}$$

where n^m is the boundary unit normal, $\mathcal{R}$ and $\mathcal{K}$ are the intrinsic and extrinsic boundary curvature ,respectively, we then have

$$L_q = \int d^4x\,\sqrt{\det\gamma}\left[\pi_{\sigma\nu}\dot{\gamma}^{\sigma\nu} + \Pi\,\dot{\Phi} - N\,\mathcal{H}\right], \tag{A.5}$$

with

$$\mathcal{H} \equiv N \left[\frac{1}{X_q} \left(\frac{1}{3} \pi^2 - \pi_{\sigma\nu} \pi^{\sigma\nu} \right) + \frac{1}{2\, G_q} \Pi^2 + \frac{X_q' \, \pi \, \dot{\Phi}}{3\, X_q} - \frac{X_q' \, \mathcal{K} \, \Pi}{G_q \, N} - \mathcal{L} \right], \qquad \text{(A.6)}$$

$$\mathcal{L} \equiv X_q \, \mathcal{R} + \frac{1}{2} G_q \, \gamma^{\sigma\nu} \, \partial_\sigma \Phi \, \partial_\nu \Phi + V_q \,, \qquad \text{(A.7)}$$

where $'$ denotes derivative with respect to Φ, and $\dot{}$ with respect to ρ.

The canonical momenta are defined by

$$\pi_{\sigma\nu} \equiv \frac{1}{\sqrt{\det \gamma}} \frac{\delta L_q}{\delta \dot{\gamma}^{\sigma\nu}} = \frac{X_q}{N} \left(\mathcal{K}_{\sigma\nu} - \gamma_{\sigma\nu} \, \mathcal{K} \right) - \gamma_{\sigma\nu} \, X_q' \, \dot{\Phi} \,, \qquad \text{(A.8)}$$

$$\Pi \equiv \frac{1}{\sqrt{\det \gamma}} \frac{\delta L_q}{\delta \dot{\Phi}} = G_q \, \dot{\Phi} + \frac{2\, X_q' \, \mathcal{K}}{N} \,. \qquad \text{(A.9)}$$

By inverting these equations, we obtain the flow equations

$$\mathcal{K}_{\sigma\nu} \equiv \frac{1}{2\, N} \dot{\gamma}_{\sigma\nu} = \frac{N}{X_q} \left[\pi_{\sigma\nu} - \frac{1}{3} \gamma_{\sigma\nu} \left(\pi + X_q' \, \dot{\Phi} \right) \right], \qquad \text{(A.10)}$$

$$\dot{\Phi} = F \left(\Pi + \frac{2\, X_q'}{3\, X_q} \pi \right), \qquad F \equiv \frac{1}{G_q - \frac{8\, X_q'^2}{3\, X_q}} \,, \qquad \text{(A.11)}$$

and then substituting these two equation into (A.6), the hamiltonian density $\mathcal{H}$ can be expressed completely in terms of the canonical momenta.

Define the hamiltonian-Jacobi functional as before

$$S[\gamma, \Phi] = \Gamma[\gamma, \Phi] + \int dx^4 \sqrt{\det \gamma} \left[Z_q(\Phi) \mathcal{R} + \frac{1}{2} M_q(\Phi) (\nabla \Phi)^2 + U_q(\Phi) \right] \qquad \text{(A.12)}$$

and solve the hamiltonian-Jacobi equation and the flow equations by adopting the new energy scale parameter μ_q defined in (4.1). We obtain the beta function

$$\beta_q = \Omega^{-1} \beta = 2\, \Phi (\Phi^2 - 1) \,, \qquad \Omega \equiv \frac{3}{5\, \Phi^2 - 2} \qquad \text{(A.13)}$$

and the renormalized dilatonic potential and coefficient of the scalar curvature

$$U_q = \Phi^{10/3} \, U = \frac{10\, \Phi^2 - 4}{\ell} \,, \qquad Z_q = \Phi \left(\Phi^2 + 2 \right) + \frac{Z_{q0} \, \Phi}{\sqrt{\Phi^2 - 1}} \,. \qquad \text{(A.14)}$$

As mentioned in section 4, if we take $Z_{q0} = Z_0$, then $Z_q = \Phi^{5/3} Z$, and the resulting c-functions transform as vectors on the Φ-space.

The formal expressions for the c-functions are somewhat involved:

$$c = \frac{1}{T} \frac{Z_q^2}{X_q} \,, \qquad \text{(A.15)}$$

$$d = \frac{1}{T} \left[\frac{Z_q^2}{3\, X_q} + \frac{Z_q'^2}{2\, G_q} - Z_q \, H - \frac{X_q' \, Z_q' \, Z_q}{3\, G_q \, X_q} + \frac{4\, X_q' \, Z_q' \, H}{G_q} \right], \qquad \text{(A.16)}$$

with

$$T \equiv \frac{-U_q}{3\,X_q} + \frac{F\,X_q'}{3\,X_q}\left(U_q' - \frac{8X_q'\,U_q}{3\,X_q}\right) + \frac{X_q'\,U_q'}{3\,G_q\,X_q} + \frac{8\,F\,X_q'^3\,U_q'}{9\,G_q\,X_q^2}\,, \tag{A.17}$$

$$H \equiv \frac{F X_q'\left(Z_q' - \frac{2\,X_q'\,Z_q}{3\,X_q}\right)}{3\,X_q}\,. \tag{A.18}$$

However, the final expressions are very simple

$$c_q = \Omega^{-1}\,c\,, \qquad d_q = \Omega^{-1}\,d\,, \tag{A.19}$$

as long as $Z_{q0} = Z_0$. Indeed, by continuity at $\Phi = 1$, we are forced to take $Z_{q0} = Z_0 = 0$.

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